A General Second Order Analog Filter to Eliminate Limit Cycles For Constant Input Signals

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Abstract -- Dither refers to a signal with small amplitude and zero mean that is intentionally injected into a system to effectively remove nonlinear distortions of the signal processed. The technique can be incorporated only when the parameters of limit cycles i.e. amplitude & frequency are known. This paper examines the determination of amplitude and frequency of limit cycle of two-dimensional electronic filters. Analog Filters consist of components such as op-amp, resistors, capacitors that exhibit non-linear characteristics. These non-linearities of electronic components leads to limit cycles in the electronic system. The limit cycles correspond to oscillations of fixed amplitude & period. The limit cycles result in marginal stability and is tolerable only if its amplitude is within some specified limits. The constant oscillations associated with the limit cycles can cause deteriorating performance or even failure of the system hardware and therefore, the limit cycle problem is an important issue that is required to be solved when dealing with analog filters. The author has used graphical technique suitable for computer graphics to determine the amplitude and frequency of limit cycles. The results are compared with the help of digital simulation technique using MATLAB 7.5.

Keywords: Multivariable system, Phasor diagram, Limit Cycle, Non-linear system.

II. LIMIT CYCLE PARAMETERS OF TWO DIMENSIONAL ELECTRONIC FILTER CIRCUITS

Considering a system consisting of 2\(^{nd}\) order electronic filters, characterized as two-dimensional multivariable systems. Transfer functions and frequency response are as mentioned below.

A. Voltage Transfer Equations of 2\(^{nd}\) order Filters

Filters are typically specified by voltage transfer equation.

\[ H(s) = \frac{V_o(s)}{V_i(s)} \]

Filters are of different orders e.g. 1\(^{st}\) / 2\(^{nd}\)/3\(^{rd}\) and other higher orders and types e.g. low pass, high pass, band pass, band stop or all pass filter. For the 1\(^{st}\) order filter the response is not ideal as the rate of decay is small therefore higher order filters are implemented to improve the response. The 2\(^{nd}\) order filters are used in the system for which the normalized transfer functions are,

2\(^{nd}\) order LPF \[ H(s) = \frac{\alpha s}{s^2 + \omega_0^2} \]

2\(^{nd}\) order HPF

2\(^{nd}\) order BPF \[ H(s) = \frac{-\alpha s}{s^2 + \omega_0^2} \]

2\(^{nd}\) order BRF

B. Analog Filters Frequency Response

The frequency response of the filters mentioned are shown in Fig.1(a, b, c, d)

C. Representation of Analog Filter System

The block diagram of a general 2x2, two-dimensional analog filter system is shown in Fig.2. Each subsystem (S\(_1\) & S\(_2\)) consists of non-linearity (N\(_1\), N\(_2\)) e.g. Saturation, relay etc. and a stable part such as analog filter connected in series. This is
the most general structural representation of two-dimensional system.

Replacing the non-linearity by its describing function (example describing function of saturation is given by Eq.1).

Describing function of the Saturation

\[
DF = N(X) = 2m \sin^{-1}(\delta/X) + (\delta/X) \left[1-(\delta/X)^2\right]^{1/2}
\]

where \(\delta\): amplitude and \(m\): slope of the saturation

Characteristics of Saturation shown in Fig.3

(a) Low Pass

(b) High Pass

(c) Band Pass

(d) Band Stop

The stable part is replaced by transfer function of filter, the phasor diagram for whole system in autonomous state is shown in Fig.4.

Figure 2. General 2×2 Electronic Filter System.

Figure 3. Characteristics of Saturation.

Figure 4. Phasor Diagram for 2×2 Analog Filter System.

For a fixed frequency \(\omega\) the angles \(\phi_1\) and \(\phi_2\) are fixed. The sides of DOBD\(_2\) correspond to subsystem-1 and that of DOBD\(_1\) represent for subsystem-2. For various values of \(R_1\) it is necessary to construct separate phasor diagrams to check the conditions for a fixed frequency. If all the quantities are normalized w.r.t the magnitude \(R_1\), a single phasor diagram (for a particular value of frequency) can be used for checking possibility of limit cycle. This diagram is called as normalized phasor diagram shown in Fig.5.

Figure 5. Normalized Phasor Diagram for Second Order 2x2 Analog Filter System.

F. Mathematical Modeling of the 2x2 Analog Filter System

The mathematical modeling of the system shown in Fig.2 is performed using Fig.5. From the Fig.5, we can derive:
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\[ OA = \frac{C_2}{R_1} = -1.0 \]

\[ OB = \]

0 \[ D_i = \frac{C_i}{R_i}, i = 1 \text{or} 2 \]

\[ BD_i = \frac{X_i}{R_i} \]

\[ AD_i = \]

In Fig. 4, C is the center of circle OBD of radius \( r=OC \). Selecting O as the origin, the co-ordinates of the points D can be determined. From triangle DCO the co-ordinates of the point C are \((0.5, -0.5/\tan \theta_L)\). The radius of the circle OC = \(0.5/\sin \theta_L\). The equation for a straight line AD is obtained as:

\[ u = (v \cot \theta_L) - 1 \quad (2) \]

The co-ordinates of the intersection points, \( D_i \) of the circle and straight line AD are obtained as:

\[ R_C^3 \cot \theta_{L_1} + \cot \theta_{L_2} + (3 \cot \theta_{L_2} + \cot \theta_{L_1})^2 - 8 \cos \ec^2 \theta_{L_1} \]

\[ u_i = (v_i/\tan \theta_{L_2}) - 1 \quad (4) \]

\[ OD_i = (u_i^2 + v_i^2)^{0.5} \]

\[ OA = -1.0 \]

\[ BD_i = [(1 - u_i)^2 + v_i^2]^{0.5}; \]

\[ AD_i = [(1 + u_i)^2 + v_i^2]^{0.5} \quad (5) \]

\[ N_1(X_1) = Y_1/X_1 = C_1/(G_1X_1) = OD_i/(G_1BD_i) \]

\[ N_2(X_2) = Y_2/X_2 = C_2/(G_2X_2) = OA/G_2AD_i \]

\[ X_1/X_2 = BD_i/AD_i \quad (8) \]

Corresponding to the value of \( N_1 \) and \( N_2 \) mentioned as describing function in Eq. 1, the ratio of \( X_1 \) and \( X_2 \) can be found from Eq (6) and (7). Similarly, the ratio of \( X_1/X_2 \) is again obtained from Eq (8). This process is repeated for different values of \( '\omega' \).

For different values of \( '\omega' \), the values of \( X_1/X_2 \) from Eq. 6 & 7 and from Eq. 8 are calculated and a graph is plotted between \( '\omega' \) and both \( X_1/X_2 \) calculated above.

E. Illustration of Graphical Method

We have considered the system shown in Fig. 2 with \( G_1 \) as the LPF and \( G_2 \) as HPF. \( G_1(s) = \frac{1.586}{s^2 + 1.414s + 1} \) and \( G_2(s) = \frac{1.586s^2}{s^2 + 1.414s + 1} \) are the transfer function of the low pass and of high pass filter respectively. The two non-linear elements \( N_1, N_2 \) are Saturation non-linearities with their parameters as \( \delta_1 = 1.5, m_1 = 1 \) and \( \delta_2 = 2, m_2 = 1 \).

For different values of \( '\omega' \), we have calculated the ratio \( X_1/X_2 \) from Eq. 6 & 7 and also from Eq. 8. The calculated values are tabulated in Table-1.

<table>
<thead>
<tr>
<th>( '\omega' )</th>
<th>( X_1/X_2 ) from Eq. 6 &amp; 7</th>
<th>( X_1/X_2 ) from Eq. 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>1.0754</td>
<td>1.256</td>
</tr>
<tr>
<td>0.45</td>
<td>1.0596</td>
<td>1.157</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0353</td>
<td>1.098</td>
</tr>
<tr>
<td>0.55</td>
<td>1.0223</td>
<td>1.046</td>
</tr>
<tr>
<td>0.6</td>
<td>0.9985</td>
<td>1.020</td>
</tr>
<tr>
<td>0.65</td>
<td>0.9852</td>
<td>0.9675</td>
</tr>
<tr>
<td>0.7</td>
<td>0.9723</td>
<td>0.9223</td>
</tr>
</tbody>
</table>

From the tabulated values we now plot a graph between \( '\omega' \) and \( X_1/X_2 \) calculated from Eq. 6 & 7 and Eq. 8. The point of intersection of both the \( X_1/X_2 \) will give the frequency of the limit cycle. The plot is shown in Fig. 6 and from the plot we get the \( '\omega' \). From the value of \( '\omega' \), the amplitude \( C_1 \) and \( C_2 \) are calculated.

From the plot, the frequency of limit cycle, \( \omega = 0.62 \). The amplitude of limit cycle calculated at this frequency are: \( C_1 = 0.5835, C_2 = 0.3185 \).

The Graphical method is also applied for other types of electronic filters with various other non-linearities and the results are tabulated in Table-2.
Table 2
Graphical Method Results

<table>
<thead>
<tr>
<th>Type of Filter</th>
<th>Type of Non-Linearity &amp; Parameters</th>
<th>ω</th>
<th>C₁</th>
<th>C₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>LPF</td>
<td>Saturation δ=1.5, m=1</td>
<td>0.62</td>
<td>0.5835</td>
<td>0.3185</td>
</tr>
<tr>
<td>BPF</td>
<td>Saturation δ=1.5, m=1</td>
<td>0.63</td>
<td>1.0567</td>
<td>0.5575</td>
</tr>
<tr>
<td>BRF</td>
<td>Saturation δ=1.5, m=1</td>
<td>0.625</td>
<td>0.4666</td>
<td>0.1839</td>
</tr>
<tr>
<td>BRF</td>
<td>Saturation δ=2.0, m=1</td>
<td>0.63</td>
<td>0.4685</td>
<td>0.2660</td>
</tr>
</tbody>
</table>

F. MATLAB Implementation

The system shown in Fig. 2, illustrated by an example mentioned in (E) has been simulated using the MATLAB 7.5. The SIMULINK block representation is shown in Fig. 7. Transfer Function represents the LPF and Transfer Function-1 represents HPF. The non-linearity N₁ is replaced by Saturation and N₂ as Saturation-1. The simulation is run to obtain the time history plots as shown in Fig. 8(a & b). From the time history plot the frequency and amplitude of the limit cycles for the analog filter system mentioned above are computed. Similarly, for other analog filters and nonlinearities the SIMULINK model is represented and time history plots are obtained from which the frequency and amplitude of limit cycles are computed as shown in Fig. 9 - 12.

![Figure 7. MATLAB / SIMULINK Model of 2-Dimensional. Electronic Filter System with Saturation Non-linearity](image)

For other electronic filters with various nonlinearities the time history plots are as shown in Fig. 9-12 using SIMULINK.
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Figure 10 (a) Output C5 of LPF with Relay Non-Linearity
(b) Output C6 of HPF with Saturation Non-Linearity.

Figure 11 (a) Output C7 of BPF with Saturation Non-Linearity.
(b) Output C8 of LPF with Saturation Non-Linearity.

Figure 12 (a) Output C9 of BRF with Saturation Non-Linearity.
(b) Output C10 of LPF with Saturation Non-Linearity.

The Table-3 shows the SIMULINK results for various filters implemented with different nonlinearities from Fig. 8-12.

III. COMPARATIVE RESULTS
The Table-4 is amalgamation of Table-2 & Table-3 and it gives a comparison between the results obtained from Graphical Method and SIMULINK simulation.

From the results we can see a slight discrepancy between the graphical and simulated results, which is a consequence only of the approximation in Describing Function Theory (i.e. the waveforms in this region are highly non-sinusoidal).

IV. CONCLUSION
The graphical method described earlier is found to be simple, intuitive and fast means to determine limit cycle. It gives quick appreciation for the system behavior using MATLAB simulation. The use of SIMULINK for oscillation prediction provides an ideal tool for comparing the accuracy of Describing Function solutions for more complicated system. The Technique can also be extended for the analysis of other electronic systems with non-linearities.

V. ACKNOWLEDGEMENT
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VI. REFERENCES


