Synchronized Phasor Measurements

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Abstract -- **The ability of the Phasor Measurement Unit (PMU) to directly measure the system state, has led to steady increase in the use of PMU in the past decade. However, in spite of its high accuracy and the ability to measure the states directly, they cannot completely replace the conventional measurement units due to high cost. Hence it is necessary for the modern estimators to use both conventional and phasor measurements together. Chi - squares test, hypothesis test and Largest Normalized residual test have been undertaken. Another way for identifying the bad data is the group elimination technique in which a group of measurements are eliminated and data is reinserted one by one to check which measurement is bad. Upon identification of bad data that bad measurement is removed. Multiple bad data is also presented.**

Keywords: State estimation, Phasor measurements, Phasor measurement unit, Real-time measurements, Bad data, Critical measurements.

I. INTRODUCTION

THE idea of using phasor measurements in state estimation is first presented in the pioneering work of Phadke *et. al* [1]. Initially it was proposed that every bus voltage and phase angle ought to be monitored by PMU which would result in a simplified state estimation. This requirement is further relaxed due to the fact that each PMU can measure not only the bus voltage phasor but also the current phasors along all lines incident to the bus. This will lead to a real-time state estimator, as opposed to the non-linear traditional state estimator which uses conventional measurements such as Voltage magnitude and Active and reactive power injection and line flows.

While the idea of using phasor measurements appears very attractive due to its advantages in state estimation, it may not be yet practical since it requires a large number of PMUs to be installed in strategic system buses which is a costly affair.

Given the impracticality of placing many PMUs to support the linear state estimation with only phasor measurements, an intermediate solution is to use phasor measurements as additional inputs to the traditional state estimation. Accuracy and network observability can be improved when phasor measurements are included with the traditional measurements.

II. BACKGROUND ON STATE ESTIMATION

Given the following measurement equation:

$$z = h(x) + e \tag{1}$$

where :

x is the state vector (size n=2N-1),

z is the measurement vector(size m< n),

h is the vector of functions ,usually nonlinear ,relating error free measurements to the state variables,

e is the vector of measurement errors, customarily assumed to have a Normal distribution with zero mean and known covariance matrix R. When errors are independent R is a diagonal matrix with values σ^2, where is the standard deviation of the measurement errors.

The maximum likelihood estimate $\hat{x}$ is obtained by minimizing the Weighted Least Squares (WLS) function:

$$J = \sum_{i=1}^{m} \{z_i - h_i(\hat{x})\}^2 / \sigma_i^2 \tag{2}$$

The minimum of a scalar J is reached by iteratively solving the so-called Normal equations:

$$G_k \Delta_k = \quad W^{-1} [z - h(x_k)] \tag{3}$$

where:

$H(E_k) =$ is the Jacobian evaluated at $x=x_k$,

$G(E_k)= H(E_k)^T W^{-1} H(E_k)$ is the gain matrix,

W^{-1} is the weighting matrix,

$\Delta x_k = x_{k+1} - x_k$, k being the iteration counter .

Iterations finish when Δx_k is within an appropriate tolerance. It can be shown that the covariance of the estimate is:

$$cov(\hat{x}) = G(E_k)^{-1} \tag{4}$$

where $G(E_k)$ is the gain matrix computed in the last iteration.

III. STATE ESTIMATION USING PMU

This section provides the modification necessary to the procedure in previous section when synchronized positive sequence voltage and current phasors are added to the measurement set. This is the process generally used when phasor measurements are to be added to the conventional measurement set. This process requires very significant modifications to the existing EMS software.

a. Estimator with phasor measurements mixed with conventional measurements

Now consider the measurement set consisting of the vector $[z_1]$ of Section 2 and a set of positive sequence voltage and current phasors $[z_2]$ obtained from PMU. The measurement error covariance matrix of the phasor measurements is assumed to be $[W_2]$. The voltage and current phasor measurements are generally obtained in rectangular coordinates, while the state vector is in polar coordinates. The error covariance $[W_2]$ corresponds to errors in polar coordinates hence, it must be transformed according to the transformation rule for converting from polar to rectangular coordinates[2].The error covariance matrix of the phasor measurements corresponding to polar coordinates $[W_2]$ is thereby transformed by the rotation matrix $[W_2']$ corresponding to measurements in rectangular coordinate.

$$[W_2'] = [R][W_2][R^T]$$

r^N_{max} $\sqrt{2K}$

The appended measurement vector [z] is obtained by adding the current and voltage phasors to the previous measurement vector $[z_1]$.

$$[z] = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \equiv \begin{bmatrix} z_1 \\ E_r \\ E_i \\ I_r \\ I_i \end{bmatrix}$$

the subscripts "r" and " i" representing the real and imaginary parts of the phasor measurements. The voltage and current phasors are nonlinear functions of the state vector.

Let the converged value of the state vector be $[E^{(2)}]$. The covariance matrix of the errors of $[E^{(2)}]$ is

$$Cov([E^{(2)}]) = [H(E^{(2)})\, W^{-1}\, H(E^{(2)})\,]^{-1}$$

where [W] is the block diagonal matrix of the two error covariances

$$[W] = \begin{bmatrix} W_1 & 0 \\ 0 & W_2' \end{bmatrix}$$

Results: The Table given below shows the true states obtained from State Estimation using PMU.

Figure 4 shows the 14-bus system w.r.t. state estimation.

TABLE 1 - TRUE STATES OBTAINED FROM STATE ESTIMATION USING PMU

Bus No.	Voltage (p.u.)	Angle (degrees)
1	1.0600	0.0000
2	1.0450	-4.5134
3	1.0700	-11.6682
4	1.0100	-11.9420
5	1.0900	-12.0409
6	1.0638	-12.0409
7	1.0567	-13.4126
8	1.0295	-7.8998
9	1.0248	-9.4097
10	1.0514	-13.3921
11	1.0569	-12.6697
12	1.0554	-12.5953
13	1.0503	-12.7527
14	1.0359	-14.1137

IV. BAD DATA PROCESSING

One of the essential requirements of using a state estimator is the ability to detect, identify, and correct measurement errors and hence get improved results. This procedure is referred to as bad data processing. Detectability of bad data depends upon the measurement configuration and redundancy.

In this paper, chi-squares test ($\div^2$) will be used to process the measurement residual to detect the bad data in measurement set. Once the bad data is detected, the largest normalized residual ( ) test will be used to identify the bad data. Multiple bad data, bad data in critical measurements and bad data in PMU will also be presented.

Chi-squares ($\div^2$) test: It can be shown that sum of the squares of independent random variables will have a chi-squares distribution, if each variable is distributed according to the standard Normal distribution. Therefore, based on the given formulation of WLS estimation method, the objective function J(x) is expected to have a distribution which can be approximated as a chi-squares distribution with at most (m-n) degree of freedom ,where m is the total number of measurements & n is the number of state variables.

It can be shown that when $x = x^{est}$ without bad data, the mean value of J(x) equals K and the standard deviation, $\sigma_{J(x)}$, equals .

where, K= m-n (m is total number of measurements and n is total no. of states) n = 2N-1 (N the number of buses in the network).

Using the statistical properties of the objective function, the following steps can be defined as the Chi-squares (÷²) test for bad data detection:

- Solve the WLS estimation problem and compute the objective function as defined by

$$J(\hat{x})=\sum_{i=1}^{m}[z_i - h_i(\hat{x})]^2/\sigma_i^2$$

  where($\hat{x}$)is the estimated state vector of dimension n.
- Check the threshold value t_J from threshold test probability function corresponding to probability p(=0.01) & m-n degree of freedom. The probability p is defined as J($\hat{x}$) t_J.
- Test if J($\hat{x}$) t_J .If yes ,then bad data will be suspected,else no bad data will be assumed to exist.

The Figure (1) shown below is the threshold test probability function which is used to calculate the threshold value used for detecting the bad data. The point of intersection of value of J(x) on x-axis corresponding to the value of probability p(=0.01) on the y-axis gives the threshold value(=29.1).

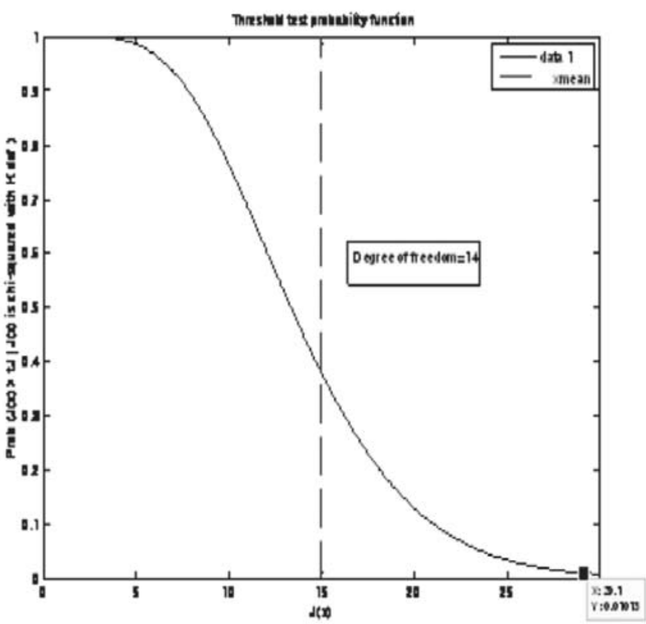


Figure 1. Threshold Test Probability Function.

Largest Normalized Residual Test: Consider the linearized measurement equation, which is used at each iteration during the numerical solution of the WLS estimation problem:

$\Delta z = H\Delta x + e$

Applying the optimization criterion, the following expression can be derived for the optimal state update:

$\Delta\hat{x} = (H^T W^{-1} H)^{-1} H^T W^{-1}\Delta z = G^{-1}H^TW^{-1}\Delta z$

where
$K = H G^{-1}H^TW^{-1}$& is called hat matrix.

Furthermore, it can be proved that the matrix K has the following property: K.H=H

Thus, the expression of measurement residuals can be derived as the follows:

$r = \Delta z - \Delta\hat{z}$
$= (I-K)\,\Delta z$
$= (I-K)(H\Delta x + e)$
$= (I - K)\, e$
$=S.\, e$

where
S = I-K & is called sensitivity matrix , which has the following property:

$S.W.S^T = S.W$

It represents the sensitivity of measurement residuals to the measurement errors. Based on the assumption that the measurement errors have Normal Distributions, the statistical properties of measurement residual are derived as:

$E(r) = E(S.e) = S.\,E(e) = 0$
$Cov(r) = Ù = E[rr^T] = S.\,E[ee^T].S^T = SWS^T = SW$

where
Ù is the covariance matrix of measurement residuals.

Hence, the normalized value of the residual for i[th] measurement can be calculated as:

$r_i^N = |r_i| / \quad {}_{ii} = |r_i|/ \quad {}_{ii}S_{ii}$

and the normalized residual vector has a standard normalized distribution, *i.e.* $r_i^N \sim N(0,1)$. It can be derived that, with enough measurement redundancy, the largest normalized residual ( ) test uses this property to identify and subsequently eliminate bad data, which involves the following steps:

- Solve the WLS estimation problem & calculate the measurement residuals:
  $r_i = z_i - h_i(\hat{x})$ i=1………m
- Calculate the normalized residuals of the measurements :
  $r_i^N = |r_i| / \quad {}_{ii} = |r_i|/ \quad {}_{ii}S_{ii}$ i=1……m
- Find the largest value r_k^N in the normalized residual corresponding to k[th] measurement;

- If $r_k^N > c$, the k^{th} measurement is identified as bad data. Otherwise, no bad data will be suspected. Here, c is chosen identification threshold (*e.g.* 3).
- Eliminate the k^{th} measurement, and repeat the state estimation.

Bad data analysis: Bad data may appear in several different ways depending upon the type, location, and number of measurements that are in error. They can be classified as:

1) *Single bad data:* Only one of the measurements in the entire system will have a large error.
2) *Multiple bad data:* More than one measurement will be in error.

Multiple bad data can therefore be further classified into three groups:

a. *Multiple non-interacting bad data:* Bad data in measurements with weakly correlated measurement residuals.
b. *Multiple interacting but non-conforming bad data:* Non-conforming bad data in measurements with strongly correlated residuals.
c. *Multiple interacting and conforming bad data:* Consistent bad data in measurements with strongly correlated residuals.

These cases had been described in detail later.

Results of bad data analysis The threshold set for the maximized normalized residual test is 3.0. The standard deviation of errors is another important element as it is used to define the covariance matrix. Due to higher accuracy of PMUs, their standard deviation of error is smaller than that of conventional measurement. Standard deviation of error for conventional measurement is set to 10^{-2} and standard deviation of error for PMUs is set to 10^{-3}. Each system is described in detail below.

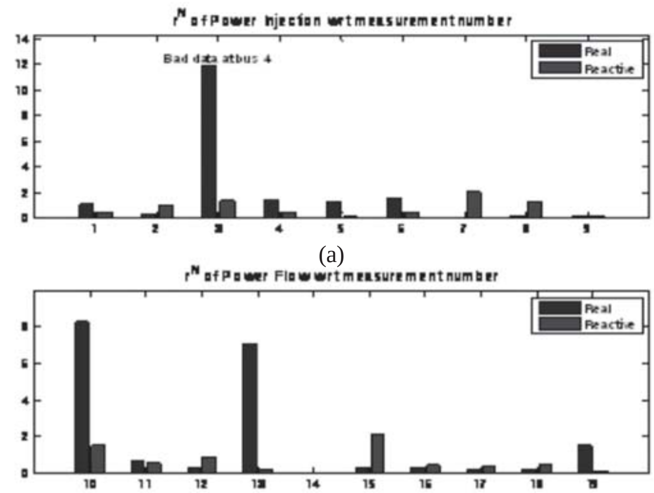
Single bad data: When there is single bad data, the largest normalized residual will correspond to the bad measurement, provided that it is not critical or its removal does not create any critical measurements among the remaining ones. This can be understood with the exercise done here.

A single bad measurement is introduced in real power injection at bus 4. The true value real power injection P_4 = -0.9420 is replaced by an error P_4 = -0.7420. Table 4 has the list of 5 Largest Normalized residues calculated using both State Estimation without PMU and State Estimation with PMU. Figure 2 and Figure 3 show all normalized residues using state estimation with and without PMU. The value of J =147.5612 which is greater than threshold of 29.1 indicates the presence of bad data.

TABLE 2- FIVE LARGEST NORMALIZED MEASUREMENTS IN CASE OF SINGLE BAD DATA (A) WITHOUT PMU (B) WITH PMU (14-BUS SYSTEM)

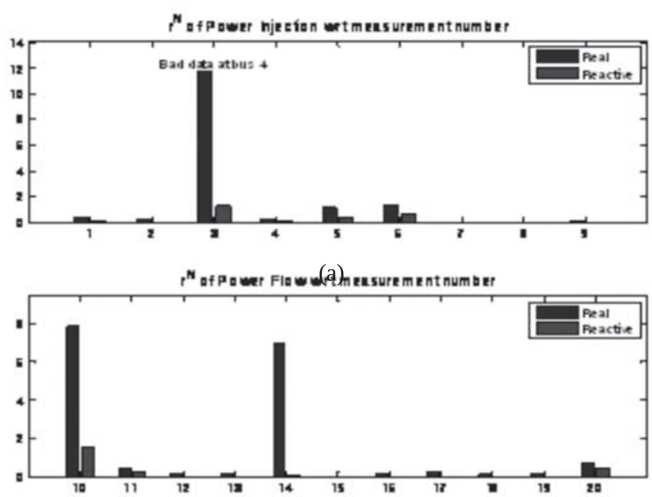
Measurement No.	Measurement	rN
3	P_4	11.9509
10	P_{2-4}	8.2790
14	P_{4-9}	7.0692
15	Q_{7-10}	2.0715
7	Q_{10}	2.0372

Measurement No.	Measurement	rN
3	P_4	11.8293
10	P_{2-4}	7.8441
14	P_{4-9}	6.9391
10	Q_{2-4}	1.5546
6	P_9	1.3476



(b)

Figure 2. Normalized residues in case of single bad data (non-critical) without PMU (14-bus system).



(b)

Figure 3. Normalized residues in case of single bad data (non-critical) with PMU (14-bus system).

TABLE 3A - MULTIPLE NON-INTERACTING BAD DATA USING STATE ESTIMATION USING PMU (14-BUS SYSTEM)

First Pass			Second Pass			Third Pass		
S.No	Measurement	rN	S.No	Measurement	rN	S.No	Measurement	rN
1	P_{2-8}	25.3768	1	P_{4-9}	16.8330	1	C_{5-6}	0.2944
2	P_{4-9}	21.4150	2	P_{4}	10.0995	2	C_{6-7}	0.2408
3	P_{8}	17.3145	3	P_{9}	10.1888	3	F_{5}	0.0789
4	P_{4}	11.3806	4	P_{8}	9.1927	4	E_{7}	0.0789
5	P_{2-4}	7.3766	5	P_{9-6}	4.9304	5	F_{6}	0.0778

TABLE 3B - MULTIPLE INTERACTING AND NON-CONFORMING BAD DATA USING PMU (14-BUS SYSTEM).

First Pass			Second Pass			Third Pass		
S.No	Measurement	rN	S.No	Measurement	rN	S.No	Measurement	rN
1	P_{4-9}	17.6198	1	Q_{4}	8.7966	1	Q_{7-10}	1.8249
2	Q_{4}	12.1829	2	Q_{2-4}	8.4984	2	Q_{10}	1.8082
3	P_{4}	11.2711	3	Q_{9}	4.6816	3	Q_{3}	0.8764
4	P_{9}	7.7666	4	Q_{9-6}	2.4642	4	Q_{11}	0.7365
5	Q_{4-9}	7.3733	5	Q_{10}	1.9787	5	Q_{13-14}	0.5075

Multiple bad data: Multiple bad data may appear in the following ways:

- *Non-interacting:*
  If $S_{ij} \approx 0$, then measurement i and j are non-interacting. In this case, even if bad data appear simultaneously in both measurements, the largest normalized residual test can identify them sequentially. This can be understood with the exercise done here.

  In this case bad data is introduced in P_{2-8} and P_{4-9}. The true value of P_{2-8} = 0.3607 and P_{4-9} = -0.2594 is changed to 0.6607 and -0.4594 respectively. The first pass identifies the bad data in P_{2-8}. After eliminating this measurement, the second pass identifies the bad data in P_{4-9}. After eliminating P_{4-9} from the measurement set third pass indicates no bad data. The table given below shows the five largest normalized values in three passes using State Estimation with PMU for 14-bus system. The value of J is equal to 1109.7 in first pass, 305.0102 in second pass and 4.3398 in third pass.

  Here, C represents the real part of current phasor, D represents the reactive part of current phasor, E represents the real part of voltage phasor and F represents the reactive part of voltage phasor.

- *Interacting, non-conforming:* If S_{ij} is significantly large, then measurements i and j are interacting. However, if the errors in measurement i and j are equal, then the largest normalized residual test may still indicate the bad data correctly. This can be understood with the exercise done here.

  In this case bad data is introduced in P_{4-9} and Q_4. The true value of P_{4-9}= -0.2594 and Q_4 = 0.0085 is changed to -0.4594 and 0.2085 respectively. The first pass identifies the bad data in P_{4-9}. After eliminating this measurement the second pass identifies the bad data in Q_4. After eliminating Q_4, third pass indicates no bad data. The table given below shows the five largest normalized values in three passes using PMU for 14-bus system. The value of J is 465.4357 in first pass, 84.8617 in second pass and 4.4171 in third pass.

- *Interacting, conforming:*
  If two interacting measurements have errors that are equal, then the largest normalized residual test may fail to identify either one. This can be understood from the exercise done here.

  In this case bad data is introduced in P_4 and P_{2-4}. The value of P_{2-4} = 0.7041 and P_4 = -0.9420 is changed to 0.5041 & -0.7420 respectively. The value of J=347.4955 indicates the presence of bad data. In first pass, the bad data is identified

TABLE 4 - MULTIPLE INTERACTING AND CONFORMING BAD DATA USING PMU (14-BUS SYSTEM)

First Pass		
S.No.	Measurement	rN
1	P_{3-11}	13.6894
2	P_{11}	10.7296
3	P_{4-9}	8.5222
4	P_{2-4}	8.0417
5	P_{3-12}	6.7698

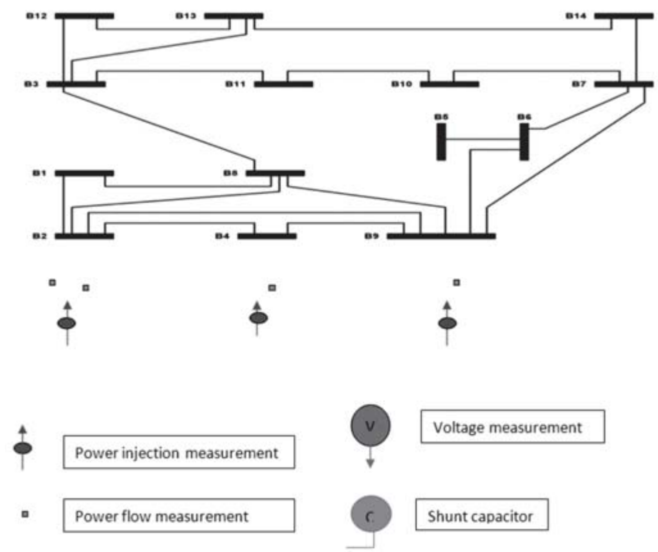


Figure 4. 14-Bus System w.r.t. State Estimation.

in P_{3-11} which clearly proved that largest normalized residual test fails to identify either of the measurement as bad data.

V. CONCLUSION

Conventional state estimation, Estimator with phasor measurements mixed with conventional measurements and addition of phasor measurements through a post-processing step is presented. The result obtained from the former method showed that they are approximately same when compared with state estimation without PMU.The slight difference in phase angles is due to round-off errors.

Various types of bad data and their causes are discussed. Threshold determination from chi-squares distribution for detection and identification of bad data through largest normalized residual test have been explained. Multiple bad data is also presented. All the results have been found correct.

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